MINI COURSES

Pierre Degond, University of Toulouse (FR)

Kinetic Modeling of Collective Dynamics and Self-Organization

Systems of self-propelled particles are widely observed in life sciences, from swarming bacteria and collectively migrating cells to flocks of birds and crowds of pedestrians. A transition from individual to collective motion is observed when the particle density increases. Collective motion is characterized by structures having much larger scales than the scales attached to each individual. These lectures intend to provide some introduction to the mathematical modelling of these phenomena. They will be split in two parts.

- *Part 1 : introduction to collective dynamics and its mathematical description : challenges and methodologies*
- *Part 2 : when geometry and topology meet collective dynamics*

Carlangelo Liverani, Università di Roma Tor Vergata (IT)

Fast-slow systems in partially hyperbolic Dynamical Systems

Fast-slow systems emerge naturally in many physical situations. An example are weakly coupled systems. A relevant question is to understand their statistical properties. While there exists a well-developed theory to investigate the statistical properties of strongly chaotic (uniformly hyperbolic) systems, little is known in the case of fast-slow systems due to the presence of "neutral directions" in which the dynamics does not mix very effectively. In the last years I have been involved in an attempt to develop a theory that applies to such systems, mainly by studying the simplest possible examples. I will describe some progresses and the obstacles of this research program. In particular, I'll explain a relation with the classical Freidlin-Wentzell theory.

INVITED SPEAKERS

Nicoletta Cancrini, Università degli Studi dell'Aquila (IT)

Mixing Time and Propagation of chaos for a Balls into Bins dynamics

I discuss the mixing time of a non reversible finite Markov chain called Repeated Balls into Bins (RBB) process. It is a discrete time conservative interacting particle system with parallel updates. I also present a quantitative estimate on the Propagation of chaos of a general class of Balls into Bins processes including the RBB process.

Alessandra Faggionato, Università la Sapienza (IT)

Hydrodynamic limit of simple exclusion processes in symmetric random environments via duality and homogenization

We consider continuous-time random walks on a random locally finite subset of $\mathbb{R}^d$ with random symmetric jump probability rates. The jump range can be unbounded. We assume some second-moment conditions (without ellipticity assumptions) and that the above randomness *is left invariant by the action of the group $G = \mathbb{R}^d$ or $G = \mathbb{Z}^d$. We then add a site-exclusion interaction, thus making the particle system a simple exclusion process. We show that, for almost all environments, under diffusive space-time rescaling the system exhibits a hydrodynamic limit in path space. The hydrodynamic equation is non-random and governed by the effective homogenized matrix D of the single random walk, which can be degenerate.*

Dmitri Finkelshtein, Swansea University (UK)

Structural properties of the mean-field expansion

The classical mean-field scheme is widely used both in statistical physics and in the study of stochastic dynamics of complex systems (individual-based models of population ecology, epidemiology, social sciences etc.). It states that when an appropriately chosen small parameter (e.g. the inverse to the number of interacting elements or a space scale parameter) tends to 0, the second and higher-order spatial correlations factorise within the dynamics, provided that the factorisation took place initially (a.k.a. the propagation of chaos property). Therefore, the spatial correlations, being expanded in the power series w.r.t. the small parameter, have an explicit leading term of the expansion, that is the product of solutions to a certain (nonlinear) kinetic equation. In the talk, I will present a new approach which describes all terms of the expansion in the small parameter through solutions to recurrent systems of linear evolution equations. The approach can be applied to a rather general class of dynamics.

Davide Gabrielli, Università degli Studi dell'Aquila (IT)

Invariant measures of the Box Ball System with independent soliton components

The Box-Ball System (BBS) is a one-dimensional cellular automaton on the integer lattice. It is related to the Korteweg-de Vries (KdV) equation and exhibits solitonic behaviour. It has been introduced by Takahashi and Satsuma, who identified conserved quantities called solitons. Ferrari, Nguyen, Rolla and Wang codify a configuration of balls by a double infinite array of integer numbers called the soliton components. Building over this codification, we give an explicit construction of a large family of invariant measures for the Box Ball System that are also shift invariant, including Markov and Bernoulli product measures. The construction is based on the concatenation of i.i.d. excursions of the associated walk trajectory. The corresponding random array of components has a product distribution with geometric marginals of parameter depending on the size of the solitons. The values of the parameters are obtained by a recursive equation.
Joint work with P.A. Ferrari.

Irene Gamba, University of Texas at Austin (USA)

Quasilinear Diffusion of magnetized fast electrons in a mean field of quasi-particle waves packets

Quasi-linear diffusion of magnetized fast electrons in momentum space results from stimulated emission and absorption of waves packets via wave-particle resonances. Such model consists in solving the dynamics of a system of classical kinetic diffusion processes described by the balance equations for electron probability density functions (electron pdf) coupled to the time dynamics waves (quasi-particles) in a quantum process of their resonant interaction. Such description results in a Mean Field model where diffusion coefficients are determined by the

local spectral energy density of excited waves whose perturbations depend on flux averages of the electron pdf. We will discuss the model and a mean field iteration scheme that simulates the dynamics of the space average model, where the energy spectrum of the excited wave time dynamics is calculated with a coefficient that depends on the electron pdf flux at a previous time step; while the time dynamics of the quasilinear model for the electron pdf is calculated by the spectral average of the quasi-particle wave under a classical resonant condition where the plasma wave frequencies couples the spectral energy to the momentum variable of the electron pdf. Recent numerical simulations will be presented showing a strong hot tail anisotropy formation and stabilization for the iteration in a 3 dimensional cylindrical model.
This is work in collaboration with Kun Huang, Michael Abdelmalik at UT Austin.

Nina Gantert, TU-Munich (DE)

The TASEP on trees
We study the totally asymmetric simple exclusion process (TASEP) on trees where particles are generated at the root. Particles can only jump away from the root, and they jump from x to y at rate $r_{x,y}$ provided y is empty. Starting from the all empty initial condition, we show that the distribution of the configuration at time t converges to an equilibrium. We study the current and give conditions on the transition rates such that the current is of linear order or such that there is zero current, i.e. the particles block each other. A key step, which is of independent interest, is to bound the first generation at which the particle trajectories of the first n particles decouple.
Based on joint work with Nicos Georgiou and Dominik Schmid.

François Golse, École Polytechnique (FR)

From the N-Body Schrödinger Equation to the Euler-Poisson System
We prove the joint mean-field and classical limits for the N-body Schrödinger equation with Coulomb potential, in the case where the Wigner measure of the initial, single particle reduced density matrix is monokinetic, i.e. is supported in the graph of a velocity field.
Work in collaboration with T. Paul.

Claudio Landim, IMPA (BR)

Large deviations for the symmetric exclusion process with weak boundary interactions
We prove the dynamical large deviations for this model and show that the quasi-potential solves a nonlinear differential equation.

Stefano Olla, Université Paris Dauphine (FR)

Quasi-static hydrodynamic limits among non-equilibrium stationary states
The quasi-static scaling limit corresponds to changes of the boundary conditions (boundary tension, temperature heat bath, density of particle reservoirs) on a time scale that is slower (i.e. larger) than the equilibrium relaxation scale of the dynamics in the bulk. In this very large time scale, the system is always very close to the global stationary state corresponding to the (time varying) tension, temperatures or densities applied at the boundaries. These

quasi-static evolutions are usually presented as idealization of real thermodynamic transformations. On the other hand they are necessary concepts in order to construct thermodynamic potentials, for example to define thermodynamic entropy from Carnot cycles. The existence of the quasi-static transformations can be seen as another thermodynamic principle that needs to be derived from the microscopic dynamics under a proper space-time scaling that we call 'quasi-static hydrodynamic limit'. We are particularly interested in studying quasi-static transformations among non-equilibrium stationary states (NESS). I will expose some results concerning quasi-static hydrodynamic limits for dynamics with diffusive behavior (like symmetric simple exclusion and anharmonic chains in contact with heat bath in the bulk and tension at the boundary) and hyperbolic/ballistic behavior (like the asymmetric simple exclusion). We also studies the large deviations from these quasi-static limits and the relation with the large deviations in the corresponding NESS.
In collaboration with Anna de Masi, Lu Xu, Stefano Marchesani.

Leonid Petrov, University of Virginia (USA)

Probability from quantum integrability
I will explain some connections between quantum integrable systems (in the sense of the Yang-Baxter equations) and particle systems. In particular, I plan to show how this leads to new reversible models playing on top of nonreversible systems. Imagine having air and vacuum in two halves of a room, and removing the separating barrier. I will explain how for very special (integrable) stochastic particle systems one can explicitly "undo" the rarefaction, and construct another Markov chain which "puts the air back into its half of the room".

Frank Redig, TU-Delft (NL)

Orthogonal polynomial duality and fluctuation fields
I will explain orthogonal polynomial duality in the context of interacting particle systems and give two applications. First is a quantified and generalized version of the Boltzmann-Gibbs principle. Second is the study of so-called higher order fluctuation fields which generalize the density fluctuation field. Using orthogonal polynomial duality, one can show that in the diffusive scaling limit these fields satisfy a recursively defined martingale problem.
Based on joint work with M. Ayala and G. Carinci.

Valeria Ricci, Università degli studi di Palermo (IT)

Non-Thermal Fusion Burning Plasma Regimes
The Ignitor project is aimed at approaching ignition conditions in Deuterium-Tritium plasmas in order to investigate the issues related to the kind of characteristics of these plasmas, following the line of compact high field opened with the Alcator and Frascati Torus programs, which were capable of producing high density plasma regimes with record confinement properties combined with high degrees of plasma purity. We present a theoretical model describing the emergence of tridimensional structures in magnetically confined plasmas which can significantly change the conditions under which meaningful fusion burn conditions can be reached. These structures can be said to consist of "captive ballooning modes", that, being localized along the radial direction, do not propagate their energy outside the plasma column.

Francesco Salvarani, Università degli studi di Pavia (IT)

Exponential convergence towards consensus for non-symmetric linear first-order systems in finite and infinite dimensions

In this talk, we consider first-order consensus systems with time-constant interaction coefficients, both in the finite and in the infinite-dimensional cases. For symmetric coefficients, convergence to consensus is classically established by proving, for instance, that the usual variance is an exponentially decreasing Lyapunov function. We investigate here the convergence to consensus in the non-symmetric case. We identify a positive weight which allows to define a weighted mean corresponding to the consensus, and obtain exponential convergence towards consensus. Moreover, we compute the sharp exponential decay rate.
The results presented in this talk have been obtained in collaboration with Laurent Boudin and Emmanuel Trélat.

Gunter Schütz, Forschungszentrum Jülich (DE)

Integrability, supersymmetry and duality for vicious walkers with pair creation

We study a system of independent random walkers in one dimension that annihilate immediately when two particles meet on the same site. In addition, pairs of particles are created randomly on neighbouring sites. For periodic boundary conditions, a duality with independent two-level systems which arises from the integrability of the model is proved. The duality function is determinantal and can also be expressed by a matrix product. We use this duality to compute the exact current distribution. For reflecting boundaries the Markov generator commutes with the generators of a subalgebra of the universal enveloping algebra of the Lie superalgebra $sl(1|1)$ and its deformations. The supersymmetry is shown to lead to a duality between an even and odd number of particles, respectively.

Fabio Toninelli, TU Vienna (AT)

Diffusion in the curl of the 2-dimensional Gaussian Free Field

I will discuss the large time behaviour of a Brownian diffusion in two dimensions, whose drift is divergence-free, ergodic and given by the curl of the 2-dimensional Gaussian Free Field. Together with Cannizzaro and Haundschmid, we prove the conjecture by B. Toth and B. Valko that the mean square displacement is of order $t\sqrt{\log t}$. The same type of superdiffusive behaviour has been predicted to occur for a wide variety of (self)-interacting diffusions in dimension $d = 2$: the diffusion of a tracer particle in a fluid, self-repelling polymers and random walks, Brownian particles in divergence-free random environments, and, more recently, the 2-dimensional critical Anisotropic KPZ equation. To the best of our authors' knowledge, ours is the first instance in which $\sqrt{\log t}$ superdiffusion is rigorously established.

Hendrik Weber, University of Bath (UK)

Regularity in time and space for stochastic porous medium equations

We prove regularity estimates in Sobolev spaces in time and space for solutions to stochastic porous medium equations. The noise considered is multiplicative, white in time and coloured in space. The coefficients are assumed to be Hölder continuous and the cases of smooth bounded coefficients as well as $\sqrt{u}$ are covered. The proof is based on velocity averaging techniques. The regularity obtained is consistent with the optimal regularity for the deterministic

porous medium equation developed recently by Gess.
This is joint work with Stefano Bruno (Bath) and Benjamin Gess (Bielefeld/Leipzig).

SHORT TALKS

Benjamin Anwasia, CMAT University of Minho (PT)

The Maxwell-Stefan equations as a formal limit of a system of Boltzmann equations for reactive mixtures

The formal limiting process that leads from the Boltzmann equation of the classical kinetic theory of gases to the Maxwell-Stefan equations will be, discussed. To this end, we consider a system of Boltzmann-type kinetic equations known in the literature as the simple reacting sphere (SRS) kinetic model under two different scalings. The first scaling describes a physical situation where the dominant role in the evolution of the species is played by non-reactive (mechanical) encounters, while chemical reactions are assumed to be very slow. This scaling leads to the Maxwell-Stefan equations. The second scaling describes a situation where both mechanical and chemical encounters proceed at the same time scale, and it leads to the reactive Maxwell-Stefan equations.

Gaëtan Cane, Université Côte d'Azur (FR)

Study of a lattice dynamic submitted to a local noise preserving the energy

In 2010, Giada Basile, Stefano Olla and Herbert Spohn published a paper where they used the Wigner's distribution to derive the kinetic limit of a "stochastically perturbed lattice dynamics". Following what they did we study the same deterministic system but submitted to a local noise preserving the energy of the system. In this talk, we will state the convergence in the kinetic time-scale of the Wigner's functional to the unique measure valued solution of some linear Boltzmann's equation. Then, we will show that this equation can be interpreted as the infinitesimal generator of a PDMP which will be study.

Leonardo De Carlo, Scuola Normale Superiore di Pisa (IT)

Mass transport in models with vorticity

Recently we studied interacting particles systems with vorticity, where we proved that the diffusion matrix in the Fick's law for the macroscopic current is given as the sum of a symmetric matrix and an antisymmetric one. In this short talk we discuss that switching on a weakly external field we obtain a symmetric mobility matrix that is related just to the symmetric part of the diffusion matrix by the Einstein relation. The proof show that this is not related to the intensity of the external field, implying that vortices in interacting particle systems can not transport mass.
Joint work with D. Gabrielli, P. Gonçalves, A. Occelli

Eduardo de Souza Böer, Universidade Federal de São Carlos (BR)

Study of $(p, N)-$Choquard logarithmic equations involving a nonlinearity with exponential critical growth

Since Choquard equations are known by its variety of applications, inspired by some recent works, we study the existence and multiplicity of solutions for the generalized version of Choquard Logarithmic equations $-\Delta_p u - \Delta_N u + a|u|^{p-2}u + b|u|^{N-2}u + \lambda(\ln|\cdot| * G(u))g(u) = f(u)$ *in* $\mathbb{R}^N$ *, where* $a, b, \lambda > 0$, $\max\{\frac{N}{2}, 2\} < p < N$, $f : \mathbb{R} \to \mathbb{R}$ *is a continuous function that behave like* $\exp(\alpha|u|^{\frac{N}{N-1}})$ *at infinity, for* $\alpha > 0$*, and* $g : \mathbb{R} \to \mathbb{R}$ *is a continuous function that has polynomial growth, with* $G(s) = \int_0^s g(\tau)d\tau$*. Using variational techniques, we guarantee the existence of a non-trivial solution at the mountain pass level and a non-trivial ground state solution. Moreover, using a version of the Symmetric Mountain-Pass Theorem, we get infinitely many solutions.*

Dimitri Faure, ENS Ulm (FR)

Averaging of semigroups associated to diffusion processes on a compact set
The problem of the averaging of diffusion processes has been extensively studied in the case we can clearly identify slow and fast variables: in the usual situation the (non-Markovian) slow variable converges in a weak sense to an effective diffusion process living in the space of the slow coordinates. In this talk we present new results on diffusion processes living in a compact set K *where the diffusion process converges in a weak sense to a continuous-time jump Markov process living on a finite subset of* K *named* K_0*, under a geometrical assumption on* K_0 *which is somehow optimal.*

Luís Simão Ferreira, CAMF-cIO Universidade de Lisboa (PT)

The Kac process: spectral gaps and entropy production inequalities
We will introduce the Kac process as a simplified model of particle collisions in a homogeneous gas, and briefly explain how Carlen, Carvalho and Loss derive a uniform bound for the generator's spectral gap, showing exponential relaxation to equilibrium. We then show that the gap for the missing 3 *interacting particle case is at least* 0.02*, giving a fully quantitative description of the gap by studying a related eigenvalue problem and conditional expectation operators on the sphere. Furthermore, we make use of Monte Carlo simulations and entropy production inequalities to estimate the value of the real gap, and discuss how adapting an argument from Villani could be used to obtain stronger results.*

Kohei Hayashi, Univeristy of Tokyo (JP)

Spatial-segregation limit for exclusion processes with two components under unbalanced reaction
We consider exclusion processes with two types of particles which compete strongly with each other. In particular, we focus on the case where one species does not diffuse at all and killing rates of two species are given by monomials with distinct exponents. We study limiting behavior of interfaces which appear by such a strong competition. Consequently, three kinds of limiting behavior of interfaces (vanishing, moving and immovable interfaces) are derived directly from our interacting particle system taking advantage of hydrodynamic limit procedure with singular limit for annihilation dynamics.

Eurica Henriques, CMAT, Universidade de Trás-os-Montes e Alto Douro (PT)

Harnack Inequalities for doubly nonlinear evolutionary equations

In the last few years many progresses were made in understanding the right form of the Harnack inequalities for the doubly nonlinear evolutionary equations given by

$$u_t - div\big(|u|^{m-1}|Du|^{p-2}Du\big) = 0, \qquad p > 1. \tag{0.1}$$

Theses equations are known in the literature to be degenerate within the range $m+p>3$, *and to be singular when* $2<m+p<3$. *The more they are becoming singular the less is known. In this talk we present several of the contributions made in deriving Harnack estimates for* (0.1), *both for the degenerate and singular cases, and present our contribution to this subject when considering* $(m+p=2 \ \wedge \ p>1)$ *and* $3-p<m+p<2$ *- very singular equations.*
Joint work with S. Fornaro, Univ. Pavia, and V. Vespri, Univ. Florence.

Michael Hott, University of Texas at Austin (USA)

On quantum Boltzmann fluctuation dynamics at the presence of a Bose-Einstein condensate

The derivation of a Boltzmann equation from first principles has been a long-standing problem. In a seminal work, Lanford showed the validity of a classical Boltzmann equation for an ideal dilute hard-sphere gas for times up to one third of the collision time. A lot of progress has been made ever since. I will explain how a cubic quantum Boltzmann equation arises within the fluctuation dynamics of a Bose-Einstein condensate, starting with the van Neumann equation for an ideal Boson gas.

Milton Jara, IMPA (BR)

Mixing time of the stochastic Curie-Weiss model

We compute with sharp precision the speed of convergence of the Glauber dynamics for the mean-field Curie-Weiss model. In particular, we show that initializing the simulation with the right density improves convergence by a multiplicative factor, but it does not modify the order of convergence.
Joint work with Freddy Hernndez.

Byron Jiménez-Oviedo, Universidad Nacional de Costa Rica (CR)

Non-equilibrium stationary hydrodynamical properties of the boundary driven Zero-Range process with long jumps

We consider the stationary zero range process with long-range dynamics in contact with infinite reservoirs. We show some properties: Hydrostatic limit and Fick's law. In the stationary case those properties are obtained from the relation between the boundary exclusion process with long jumps and the boundary zero range process with long jumps.

Julian Kern, Ecole normale supérieure de Lyon (FR)

Rough super-Brownian motion in spatially correlated environments

In a recent article, Perkoswki and Rosati proved the convergence of a rescaled branching random walk in a random environment (BRWRE) to a so-called rough super-Brownian motion. In their work, they concentrated on an iid environment converging to Gaussian spatial white noise. Based on their insights, we study how the BRWRE scales when the random environment exhibits spatial correlation. In this talk, we will extend the scaling result to a class of

Gaussian fields with weak spatial correlation. Furthermore, we will discuss what we should expect when the correlations are stronger.